1.0 Structure of Surface Altering

Surface Altering is based on a sequence of transformations applied to the geometry of the input surface. In this document, these steps are reviewed separately for 2D and 3D scenarios. Each step is solved using Bound Optimization BY Quadratic Approximation (BOBYQA) developed by Powell [1] algorithm implemented in NLopt library.

2.0 2D Surface Altering

In two-dimensional analyses, a non-circular surface in its simplest form can be described as a linear spline curve. Coordinate values of control points will form the optimization input. As an example, Figure 1 illustrates a surface with 7 control points, yielding to 14 input variables to define the x and y coordinates of the 2D surface. The geometry of the surface can be altered by modifying these coordinates. SAO offers a systematic set of steps to perform this alteration to minimize the factor of safety such that it satisfies geometrical convexity and maintains the sequence of control points. These steps are repeated in multiple iterations until convergence criteria is met. A key consideration in SAO is to realize the entire surface geometry, such that changing coordinates of one control point affects the other points, which keeps the surface convex and ordered.

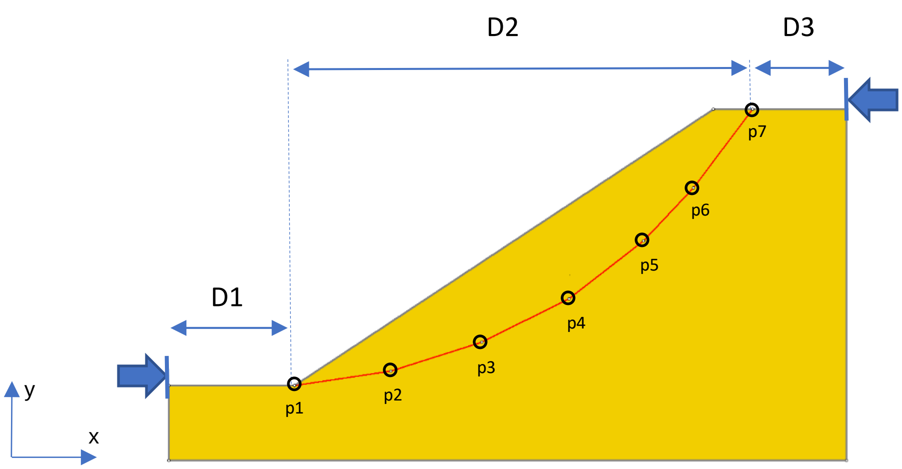

Figure 1: A linear spline 2D slip surface

2 A) Compression and Expansion with End Points

Firstly, the two end points can be translated within a defined limit to the left or right. Since these points are restricted to the slope boundary, only one of the x or y-coordinates of each point can be modified as the optimization input parameter. For non-vertical slopes, the x-coordinates of the two end points (p_1 and p_n) for a surface with n points are considered as optimization variables. When translating the left end-point (p_1), the right end-point (p_n) is considered fixed, and all the in-between points are shifted in relation to the left node displacement. After displacements are applied on each point, the same translation is applied for right-end and in-between points, considering the left point fixed. Equation 1 defines the displacement applied on point P_i, when end-point P_a is displaced a distance of d_a in the x-direction, and end-point P_b is fixed. x_a, x_b, and x_i represent the x-coordinate of end points P_a, P_b, and point P_i.

$$d_i = d_a \times \frac{x_i - x_b}{x_a - x_b} \tag{1}$$

This initial step in SAO gradually modifies the location of the slip surface through multiple iterations, while preserving the order between the control points. In the presence of vertical slopes, input variables may represent displacement in the y direction. Figure 2 illustrates several scenarios that may occur with vertical slopes. In each case, depending on the location of the end-point and direction of displacement (positive or negative), displacement will be applied in either the x or y direction.

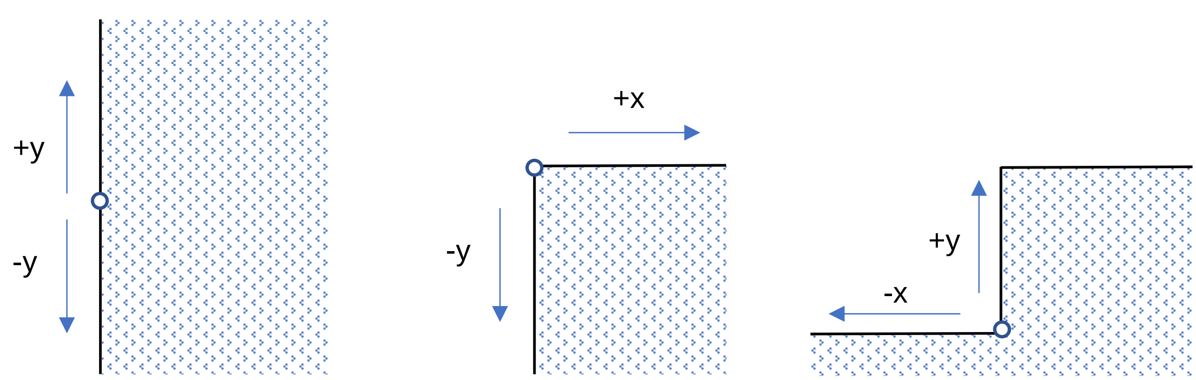

Figure 2: Different cases in displacement of an end point in presence of vertical slope

The boundary constraints for the end-point displacements can be determined based on the distance to slope limits. For example, in Figure 1, the left point is limited to D1 displacement to the left, and maximum D2 + D3 to the right. Our experiments show that introducing an extra constraint defined as a fraction of surface width (see D2 in Figure 1) results in a steadier progress of SAO. Then, static left boundary constraints for left point in Figure 1 can be defined as $\min(\mathrm{D1}, \alpha\mathrm{D2})$.

2 B) Compression and Expansion with Internal Points

In this step, a pair of control points is selected from internal points. An internal point is any surface control point, excluding the end points. Two sets of transformations will be applied to the surface geometry with respect to this control pair. In the first set, the control pair and all the points in between them will remain fixed (Figure 3, where P_3 and P_5 are selected as a control pair). This pair plus P_4, which is between the pair, remain fixed. Displacement will be applied on the two end points. The remaining control points outside of the fixed region relocate proportional to the nearest end-point displacement (see displacement of P_1, P_2 *and* P_6 in Figure 3. Equation 1 can be used to compute displacements with respect to nearest fixed point.

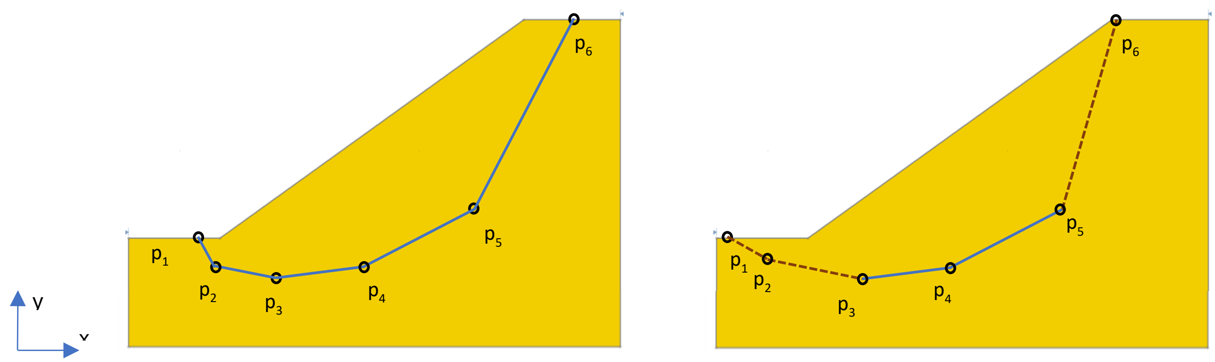

Figure 3: Scaling with respect to fixed points

In the second set of transformations, the two end-points remain fixed and the x-coordinate of the internal control pair is the optimization parameter. The pair of control points can move on a straight line passing through them, e.g. see the dashed line passing through P_3 and P_5 in Figure 4-(a). All the internal points will be displaced proportional to the new location of the two internal control points, e.g. see Figure 4-(b) where P_4 is displaced proportional to the new location of P_5.

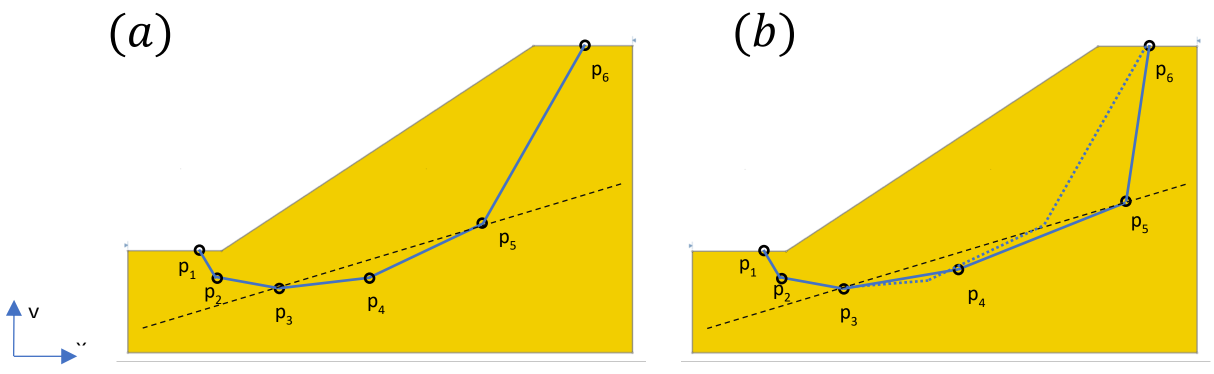

Figure 4 (a): Control pair move along a straight line; (b) Scaling of internal points proportional to new location of control pair

Since there are multiple ways to select the two internal points, this step can be repeated for several combinations.

2 C) Modifying Curvature

Curvature of the slip surface can be modified by moving the inner spline control points along positive or negative in y-direction. Optimization input parameters for a surface with n control points would be $n-2$, with each parameter representing the displacement of the y-coordinate of one control point excluding the first and last. The displacement for each point is constrained in two levels. The first level, known as static boundary constraint, is defined such that each varying control point remains within slope boundaries and under a straight line connecting the first point to the last one (Figure 3).

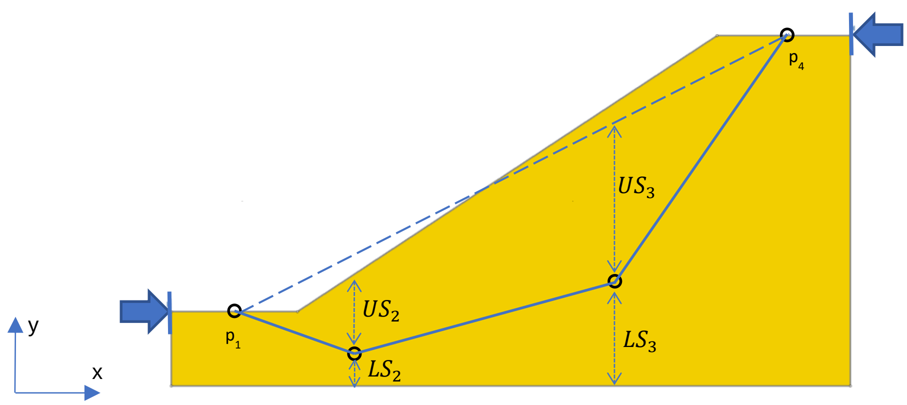

Figure 5: Static boundary constraints in step B of 2D Surface Altering

US_i and LS_i in Figure 3, represent upper and lower static bounds defined for control point i. Upper static bound for each control point is defined such that the point remains within slope geometry (US_2 in Figure 3). The other factor used to set upper static bound is to keep the point under a straight line connecting the two end-points of the surface (US_3 in Figure 3). Lower static bound is defined such that the control point remains within slope geometry (l_2 and l_3). In Slide software, we introduced an extra constraint based on slip surface width. This would limit upper or lower displacement for any point not to exceed 1.5 × [distance between two end points]. This additional constraint can improve convergence behavior of SAO.

A second level of constraint, referred to as dynamic boundary constraint, needs to be applied to enforce convexity of the slip surface. Non-convex slip surfaces can result in negative tensions at base of the slip surface and output nonsensible failure factor of safety. Therefore, in slope stability analysis, it is important to maintain convexity of the slip surface through surface altering transformations. The method used to determine dynamic boundary constraints is similar to the method adapted in [2].

Vertical displacement for each control point is applied in a sequence, starting from one of the two end points of the slip surface. Dynamic boundary constraints cannot be defined *a priori* to displacements applied to previous control points. Consider the slip surface in Figure 4, where displacements will be applied from left to right. Upper dynamic constraint for control point P_i is defined based on the locations of the previous point, P_{i-1}, and the last point, P_6. Therefore if a previous point, P_{i-1}, moves up toward positive y-direction, dynamic upper bound will shrink to zero. Upper dynamic constraint for point 4 in Figure 4 is displayed as UD_4. After determining the dynamic constraint, the point displacement, as an optimization input parameter, needs to be adjusted accordingly,

$$d'_i = d_i \times \frac{D_i}{S_i} \tag{2}$$

Where d_i is the input displacement and d'_i is modified displacement to be applied to point P_i. D_i and S_i represent dynamic and static boundary constraints. Upper dynamic bound should remain non-negative, therefore if the location of the points is such that UD_i is determined negative, y-coordinate of point P_i will be changed such that upper dynamic bound is evaluated as zero.

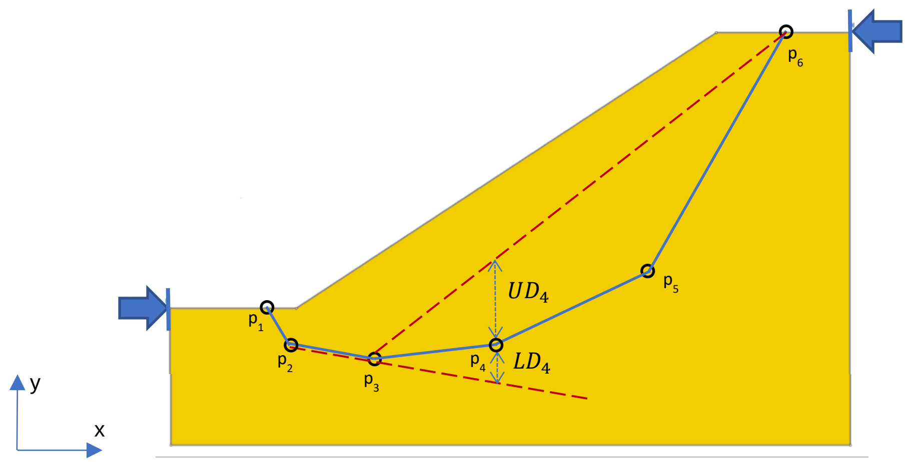

Figure 6: Dynamic boundary constraints used to maintain convexity while altering the surface curvature

Lower dynamic bound for point P_i is determined by the line passing through two points before that, e.g. in Figure 4, where displacements are applied sequentially to points P_1 to P_6, LD_4 represents lower dynamic boundary constraint for point P_4 and is determined by a line passing through P_2 and P_3.

Due to the sequential nature of displacements applied on a surface point, each control point will be further constrained compared to the previous points. In other words, a desired displacement for control points close to the end of the sequence may never be achieved due to enforced dynamic constraints. To diminish this induced constraining effect, different solutions exist. One approach is to simply repeat step B in SAO twice per iteration, once from left to right and once vice versa. An alternative approach is to partition control points into a coarse and a fine set. Then apply displacements in the following order {c_1, c_2, …, c_m, f_1, … , f_n}, where c_i and f_j represent coarse and fine points.

Steps A, B, and C described above will complete one iteration in 2D surface altering. Transformations in steps A and B are repeated until no significant difference in the input geometry or output factor of safety is achieved. For models with thin weak layers, an additional step can be introduced to improve SAO results (Section D).

2 D) Weak Layer Snapping

In presence of geology structures with weak layers, there is a good chance that the critical slip surface passes through the weak layer. Thin structured weak layers introduce the extra challenge of finding a proper set of transformations applied to the slip surface, such that it extends through the weak layer. In presence of weak layers, an additional step can be performed within step C. Going through each internal control point in step C, where the y-coordinate of the control point is altered, an additional displacement can be applied such that the control point moves inside the weak layer. If this additional displacement yields to a smaller factor of safety, then the change will be accepted.

3.0 References

1. M. J. Powell, "The BOBYQA algorithm for bound constrained optimization without derivatives," University of Cambridge, Cambridge, 2009.
2. Y. Cheng, "Location of critical failure surface and some further studies on slope stability analysis," *Computers and geotechnics,* vol. 30, no. 3, pp. 255-267, 2003.