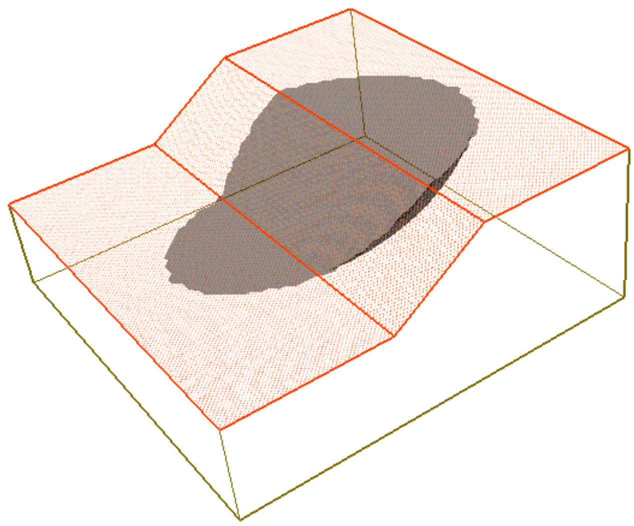
Slide3 – 3D Limit Equilibrium Slope Stability Overview

Introduction

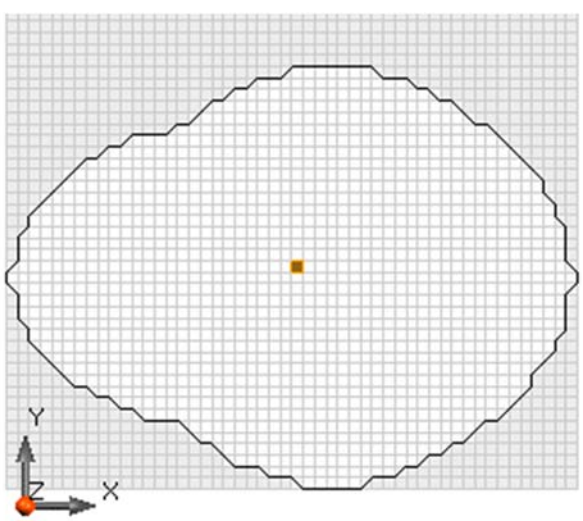
Three-dimensional limit equilibrium slope stability analysis is simple in concept, and directly analogous to 2-dimensional methods.

- In 2D a sliding mass is discretized into vertical slices
- In 3D a sliding mass is discretized into vertical columns with a square cross-section

The 2D methods of slices (Bishop, Janbu, Spencer and Morgenstern-Price (GLE)) which are based on satisfying force and/or moment equilibrium, can be extended to a 3D method of columns, where forces and moments are solved in two orthogonal directions. Vertical forces determine the normal and shear force on the base of each column.



Simple 3D slope model with 3D spherical failure surface



Plan (top) view of sliding mass discretized into square columns

Although 3D limit equilibrium slope stability analysis using vertical columns is simple in concept, it is not so simple in practice to implement efficiently and accurately. The 3D method faces many obstacles not encountered in 2D, not the least of which is how to search efficiently for unknown critical 3D slip surfaces. Furthermore, issues which are problematic in 2D slope stability analysis (e.g. how to deal with tensile forces) become magnified in 3D analysis.

Limitations of earlier methods

Early numerical methods proposed for 3D limit equilibrium slope stability computation were subject to several constraints such as:

- Assumed sliding direction
- Assumed plane of symmetry
- Transverse force and/or moment equilibrium not satisfied
- Local coordinate systems required
- Simple search methods for critical surfaces (e.g. spherical, planar)

Results were satisfactory for symmetrical 3D problems, but not for more complicated asymmetrical slopes. Early use of 3D slope stability methods was often used for back analysis of known failures, rather than searching for critical failure surfaces.

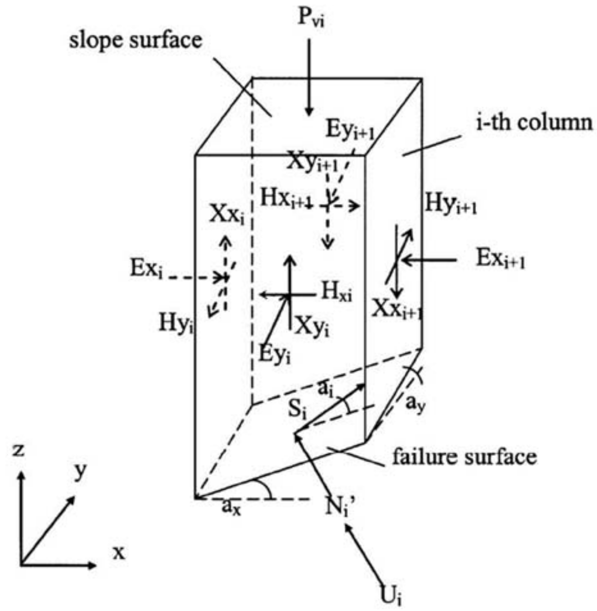
Improved 3D Methods

Significant improvements to 3D slope stability were proposed by Huang, Tsai and Chen (2002) and further extended by Cheng and Yip (2007). Improvements include:

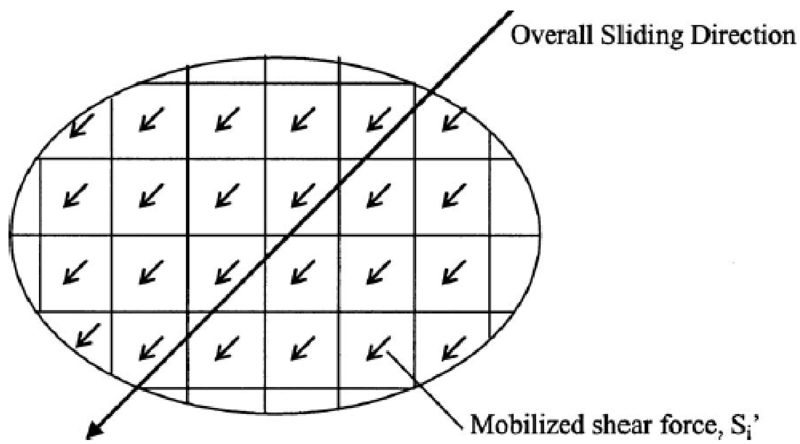
- Force and moment equilibrium in 2 orthogonal directions
- Unique sliding direction is solved for rather than assumed
- 3D system of equations is statically determinate

The forces acting on a typical column are shown below.

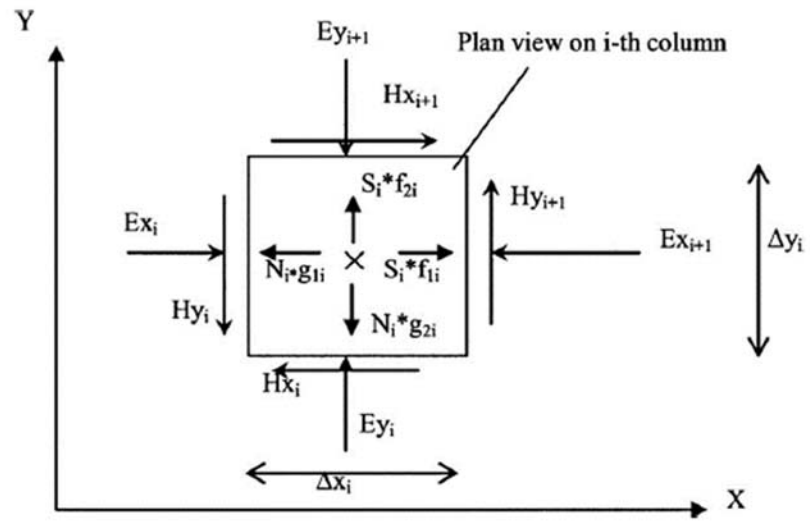
- N, U = effective normal force and pore pressure force on column base
- S = mobilized shear force on column base
- a = sliding direction
- E = intercolumn normal forces
- X = intercolumn vertical shear forces
- H = intercolumn horizontal shear forces
- P = vertical external force
- W = column weight



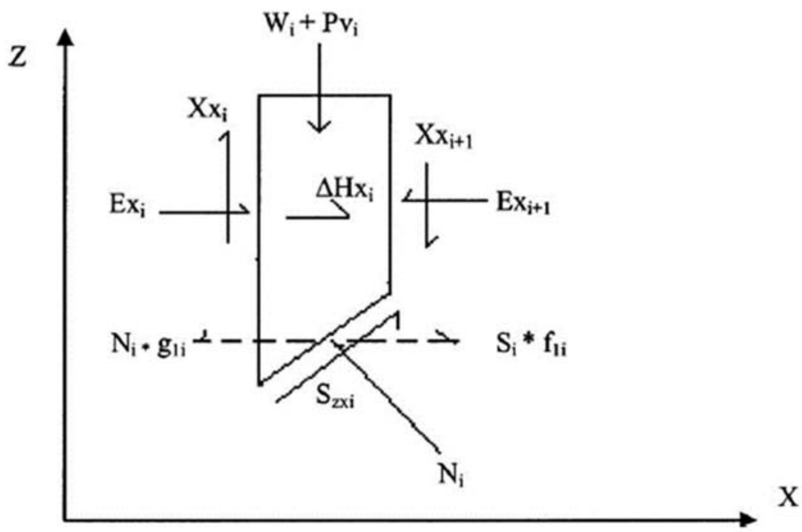
Three-dimensional view of forces acting on column



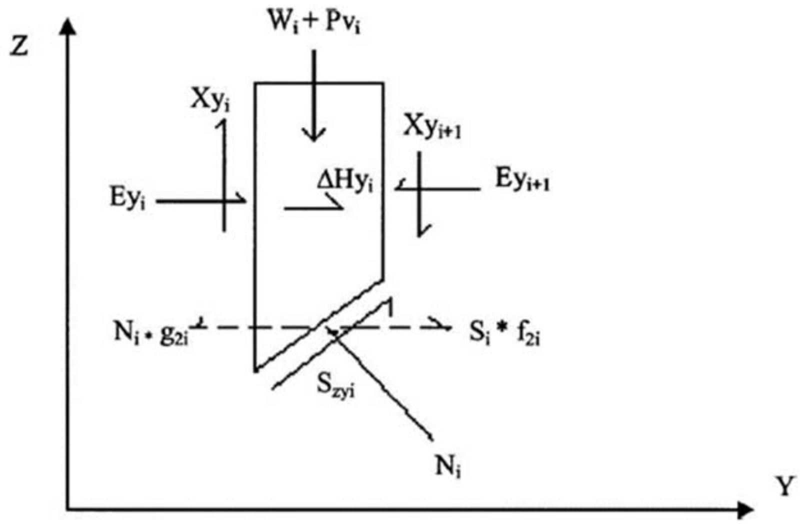
Unique sliding direction for all columns (plan view)



Force equilibrium in x-y (horizontal) plane



Horizontal force equilibrium in the x-direction for a typical column



Horizontal force equilibrium in the y-direction for a typical column

The main equations used to obtain the 3D safety factor are summarized below. For further details see Cheng and Yip (2007).

NOTE:

- The X-Y plane is the horizontal plane
- The Z-axis is the vertical direction
- f1, f2, f3 and g1, g2, g3 are unit vectors in the direction of Si and Ni respectively

Let's first consider vertical force equilibrium (z-direction) of a single column. For the ith column:

$$\sum F_z = 0 \rightarrow N_i g_{3i} + S_i f_{3i} - (W_i + P_{vi}) = (Xx_{i+1} - Xx_i) + (Xy_{i+1} - Xy_i) \quad (1)$$

The base normal and shear stresses can then be expressed as:

$$N_i = A_i + B_i S_i; \quad S_i = \frac{C_i + (A_i - U_i)\tan\phi_i}{F\left[1 - (B_i \tan\phi_i / F)\right]} \quad (2)$$

Where:

$$A_i = \frac{W_i + P_{vi} + \Delta Ex_i \lambda_x + \Delta Ey_i \lambda_y}{g_{3i}}; \quad B_i = -\frac{f_{3i}}{g_{3i}} \quad (3)$$

Overall force and moment equilibrium in the X and Y directions is given by the following equations.

Overall force equilibrium in x-direction gives:

$$-\sum Hx_i + \sum N_i g_{1i} - \sum S_i f_{1i} = 0 \qquad (4)$$

Overall moment equilibrium in the x-direction gives:

$$\sum (W_i + P_{vi} - N_i g_{3i} - S_i f_{3i}) RX + \sum (N_i g_{1i} - S_i f_{1i}) RZ = 0 \qquad (5)$$

where RX, RY, and RZ are lever arms to the moment point.

Overall force equilibrium in y-direction gives:

$$-\sum Hy_i + \sum N_i g_{2i} - \sum S_i \cdot f_{2i} = 0 \qquad (6)$$

Overall moment equilibrium in the y-direction gives:

$$\sum (W_i + P_{vi} - N_i g_{3i} - S_i f_{3i}) RY + \sum (N_i g_{2i} - S_i f_{2i}) RZ = 0 \qquad (7)$$

Equations for directional factors of safety Fx, Fy, Fmx, Fmy can be determined. We solve for when Fx=Fy=Fmx=Fmy or rewritten:

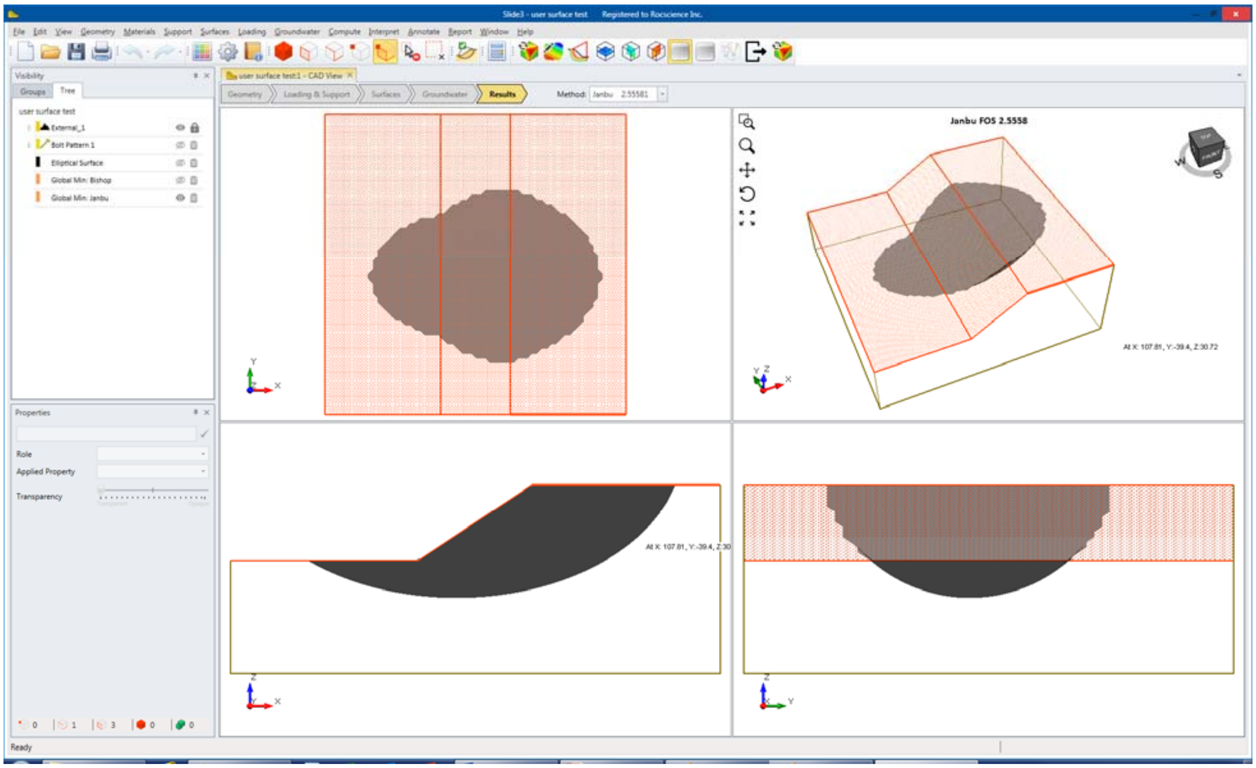
- Fy-Fx=0
- Fmx-Fy=0
- Fmy-Fx=0

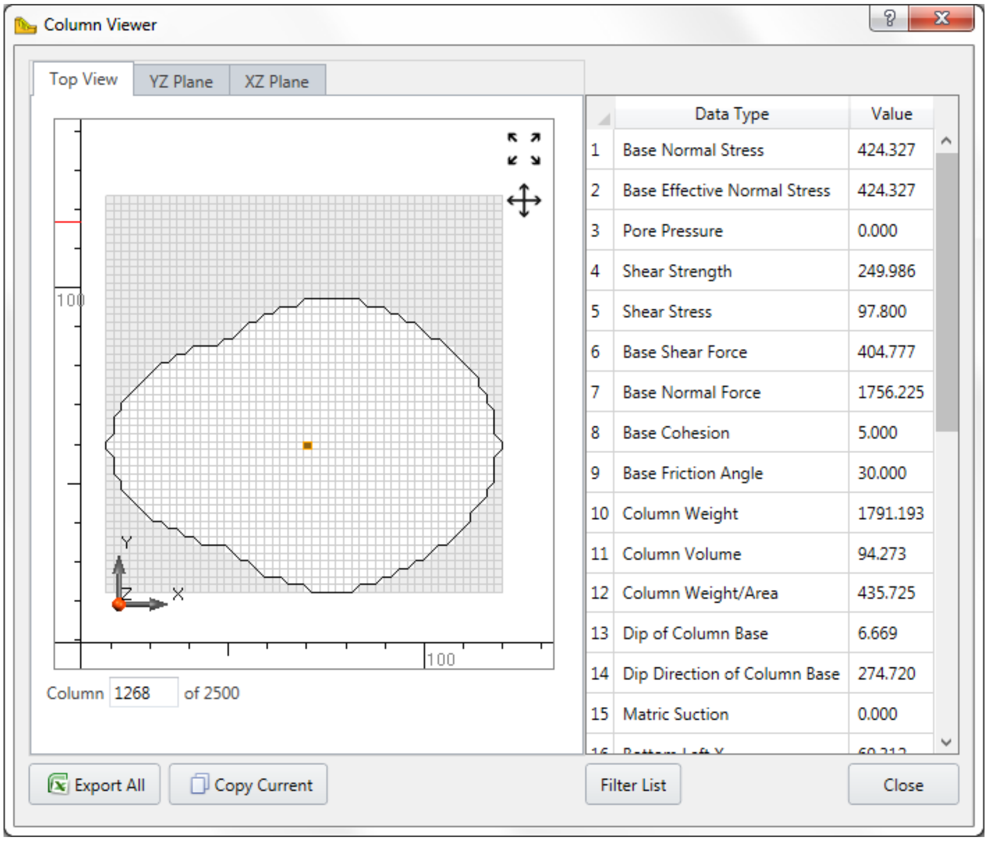
We then find the values of F, lamdax, lamday, aprime (sliding direction) that satisfy these 3 equations. The value of F is the overall 3D safety factor for a given 3D slip surface.

3D slope stability using Slide3 2017

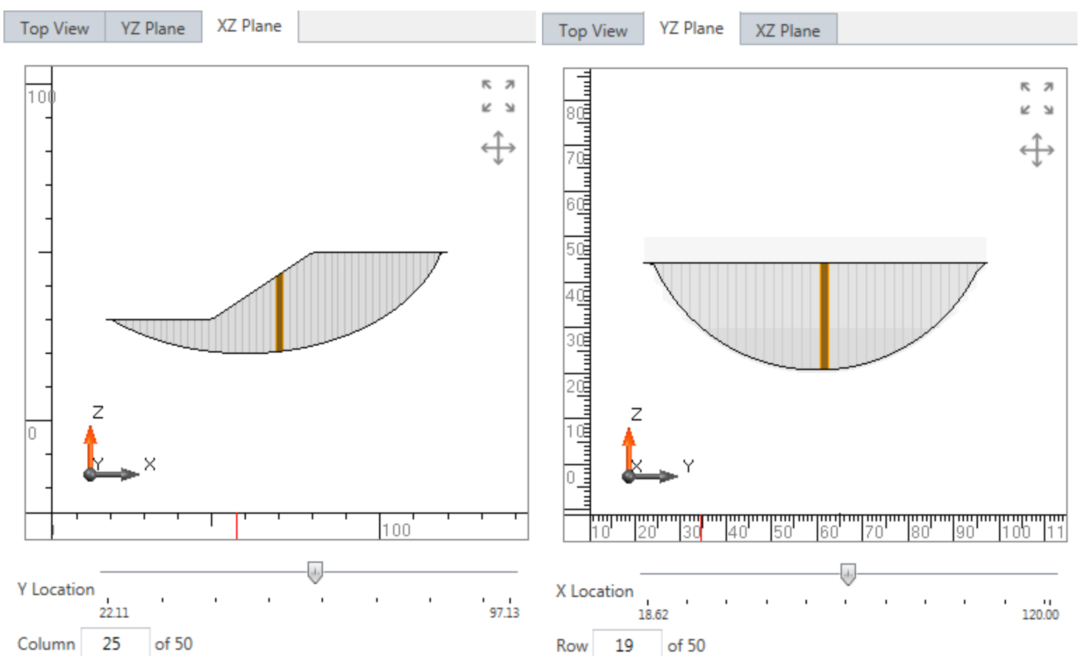
Slide3 2017 uses the general formulation of Cheng and Yip (2007), with further improvements, including:

- Efficient solver for 3D equilibrium equations
- Any failure criteria can be used (not limited to Mohr-Coulomb)
- Fast search methods for general 3D slip surfaces
- Powerful geometry modeling and data interpretation features





Column data viewer



Column viewer (vertical section views)

References

Huang, C.C., Tsai, C.C., Chen, Y.H., 2002. Generalized method for three-dimensional slope stability analysis. J. Geotech. Geoenviron. 128 (10), 836–848.

Cheng, Y., Yip, C., 2007. Three-dimensional asymmetrical slope stability analysis extension of Bishop's, Janbu's, and Morgenstern–Price's techniques. J. Geotech. Geoenviron. 133 (12), 1544–1555.